INTERTEMPORAL PRODUCTION FRONTIERS: WITH DYNAMIC DEA

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To
W.W. Cooper

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PREFACE

Our intention with this book is to extend the efficiency literature to the case of intertemporal models. We do this in steps. First, we introduce static network models which will serve as building blocks for our intertemporal budgeting models and our dynamic models. Next, we devote two chapters to productivity measurements, which we think of as comparative static models. Intertemporal budgeting models and dynamic models are taken up after that.

Each chapter, except Chapter One, contains an empirical application. These applications are coauthored with colleagues and students; thanks are due to Runar Brännlund, Yijan He, Julius Horvath, Pontus Roos, Jerry Whittaker and S. (Lek) Yaisawarng. We would also like to thank Dale Boisso and Kathy Hayes for graciously sharing their data on Illinois municipalities with us. Two of the applications are already published, namely: "Environmental Regulation and Profitability: Applications to Swedish Pulp and Paper Mills," *Environmental and Resource Economics* 6: 23-36, 1995, (Section 2.5) and "Productivity and Quality Changes in Swedish Pharmacies," *International Journal of Production Economics* 39: 137-144, 1995, (Section 3.5). We are grateful to Kluwer Academic Publishers and Elsevier Science for kindly allowing us to reproduce these publications here.

During the summer 1995 we spent a very enjoyable two months at the Center for Economic Studies (CES) at the University of Munich.

We are very grateful to Hans-Werner Sinn for providing us with this opportunity. During our stay, we gave a series of lectures based on an earlier draft of this monograph. Thanks to our vigilant students at CES, the current version has been purged of many errors, and is we believe, much improved. Many thanks as well to the wonderful CES staff, the CES soccer team and Gerlinde Sinn. They made our stay both "gemütlich" and productive.

Mariann Baratta has prepared the manuscript, including graphics. Her professionalism and patience in dealing with transatlantic revisions made this project possible.

Finally, thanks are due to Bert Balk for his keen eye.

1

INTRODUCTION

The main goal of this monograph is to generalize our earlier work on production frontiers and performance measurement to the intertemporal case. Analysis of multi period data using activity analysis models is already widespread, of course. One obvious example is our own work on measurement of productivity using activity analysis models to construct Malmquist type productivity indexes. Although many periods are analyzed, and the change in technology over time is measured, these represent a fairly limited type of intertemporal models – one is essentially comparing a series of static models. There is no interdependence or connection over time among inputs, outputs or technology. This type of analysis is the analog of comparative statics.

Our ultimate goal is to develop a fully dynamic model which can be implemented using activity analysis models. In fact, the working title of this book during its construction was 'Dynamic DEA.' This monograph includes a series of models – starting with the static and comparative static – which systematically introduce various types of interdependence in technology over time. The key idea used to introduce connections or interdependence here is the notion of a network. Although introducing a network also introduces complexity (and flexibility), the models are still readily computable using the standard programming techniques familiar to those who work in the area of performance measurement. In fact, we include an activity analysis representation of each intertemporal model introduced in the monograph.

The reader expecting a book on dynamic programming will be disappointed. The reader hoping to discover new ways to use familiar techniques will, we hope, be well rewarded. We also hope to appear to those who are interested in thinking about how to enrich our models by going inside the 'black box' of technology.

1.1 MOTIVATION

In the preface to Shephard and Färe (1980), the authors ask themselves how it is that one would go about building a large ship in an efficient way. Their work was funded by the Office of Naval Research and they had actually visited a shipyard to get a snapshot of the process. One of the key issues was that of timing. Clearly the final output would not appear until long after inputs were first applied to process? But what was the best way to go about the actual building of the process?

Shephard and Färe (1980) proceeded to address these questions as production theorists. Here we take a different tack, and take up these questions from a practitioner's point of view. Our goal is to derive models that can be implemented using simple tools, in particular the activity analysis models commonly employed in the DEA or 'nonparametric' efficiency measurement literature.

1.2 CONNECTION TO EARLIER WORK

This monograph is the outgrowth of our earlier work. As mentioned above, it was motivated by Shephard and Färe (1980). As the reader will immediately recognize, however, it is more closely related to Färe, Grosskopf and Lovell (1994) *Production Frontiers* in 'style' and content. The 'style' is that of a practical manual for practitioners, yet with axiomatic underpinnings that are pure Shephard. As in Färe, Grosskopf and Lovell, we continue to use the distance function as a key theoretical description of technology. However, here we focus much more on the computational form based in activity analysis or linear programming models of technology. This allows us to easily model the 'new' feature of technology, namely the notion of a network. This allow us to go inside the 'black box' of technology and explicitly consider intermediate products, for example.

In order not to end up with an engineering manual, we also rely heavily on our previous work on budget constrained models. In Färe and Grosskopf (1994) *Cost and Revenue Constrained Production*, we extend Shephard's earlier work on 'indirect' production. These models introduce budget constraints or revenue targets to the optimization problem. The interesting feature of these models for this monograph, is that this allows for reallocation. Here we extend these models to allow for reallocation of the budget over time. Again, these models are presented as activity analysis problems.

Although this monograph is clearly related to our earlier work, we have attempted to make this a self-contained document. That means that there will be a bit of repetition from earlier work. However, this is intended to be as user-friendly as possible, therefore the level of technical detail is kept to a minimum. For those seeking more technical detail or related work, each chapter contains a brief section on related literature.

A departure from our earlier books is the inclusion of 'real' empirical examples at the end of each chapter. These are intended to illustrate the material in the chapter. In addition, they introduce a generalization of that material.

1.3 PRODUCTION AND TIME

In this section we provide a heuristic road map to the monograph. We do so by illustrating schematically how we build time into our models of technology.

We begin at the beginning – namely with the familiar static model of technology. The basic axiomatic structure is introduced in Chapter 2, and we show that the activity analysis model is consistent with those axioms. The basic idea of the static model is illustrated in Figure 1.1. Here we have inputs, x, being employed to produce outputs, y. The transformation process is represented by the circle labelled P. In the static model, that process is treated like a black box – we do not explicitly model how inputs are transformed into outputs. This is clear from the standard linear programming or activity analysis problem used to construct technology and gauge performance: technology is constructed as convex combinations of observed input and output data. Only minimal structure is imposed in order to model minimal regularity conditions such as disposability, etc.

Chapter 2 seeks to peek inside the black box of the static technology by introducing the idea of a network. This is illustrated in Figure 1.2 for a simple case with three 'nodes' which produce inter-

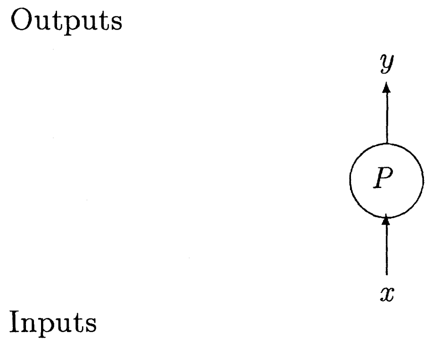


Figure 1.1 The Static Technology

mediate outputs. For example, the final output y might be finished doors with frames. Node one might produce the actual doors, while node two produces the frames. Node three assembles the doors and frames. Note that nodes one and two could also deliver their production as final products for spare parts or repair, for example. As we shall see, this simple structure can eventually be modified to allow for intertemporal production.

Chapter 3 introduces time, although in a static framework. This is the framework used in computing Malmquist productivity. One might think of this framework as a comparative static framework: we are comparing static technologies, which happen to change (exogenously in this framework) over time. This general idea is illustrated in Figure 1.3. Here we see a series of static technologies: inputs from period t are used in technology P^t to produce outputs in period t. A similar process occurs in the next period, $t+1$. The Malmquist productivity index discussed in Chapters 3 and 4 seeks to compare these static technologies, *ex post.* Notice that there is no connection between the technologies or the inputs and outputs

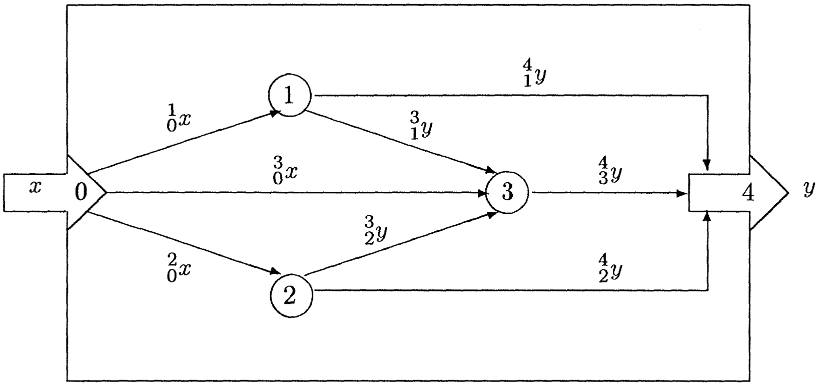


Figure 1.2 The Network Technology

across periods, i.e., we have a strong type of time separability in these models.

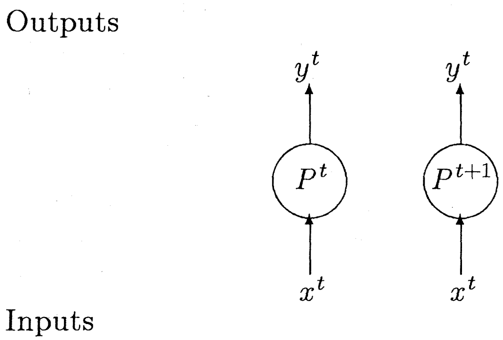
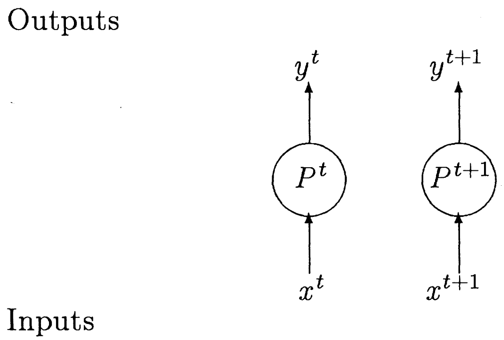


Figure 1.3 The Comparative Static Technology

Chapter 5 introduces the notion of intertemporal budgeting, which is the generalization of the static indirect models developed in Färe and Grosskopf (1994). Here a budget constraint is introduced, but the budget constraint is defined over more than one period. This provides a connection between production in each period. This idea is illustrated in Figure 1.4, where we have two periods, with a budget constraint defined over both periods. Notice that the technologies are still in some sense, static, i.e., there is no connection between the inputs, outputs or technologies over time. There is, however, a link through the budget constraint. As we shall show, this implies that total output over the two periods can be enhanced by reallocating inputs over time. Notice that instead of a strict *ex post* measure of performance, we now can use these models for *ex ante* planning.



$$w^t x^t + w^{t+1} x^{t+1} \leqq C$$

Figure 1.4 The Intertemporal Budget Model

The final chapter of this monograph takes up what we refer to as dynamic production models. Here the idea is that time plays a direct role, and that decisions/production are connected over time. The interconnection over time is modeled by introducing what we refer to as the product technology. Figure 1.5 illustrates. Here we have two periods and two technologies with period-specific inputs and outputs. The difference between Figure 1.5 and the earlier figures is that there is a connection between the technologies, represented by the horizontal arrows connecting the circles. One may think of these as representing intermediate goods that are produced in one period and employed in the next period. Notice the close resemblance between Figure 1.2 and Figure 1.5: they both have a network structure with interconnected nodes.

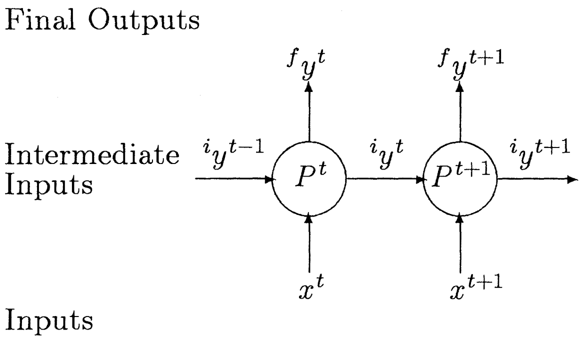


Figure 1.5 The Dynamic Model

One could also, of course, introduce something like the budget constraint from Figure 1.4 to allow interconnection among the inputs over time. Chapter 6 includes a variation on this theme, namely the idea of storable inputs.

This brief overview was designed to show the basic organization of the text and to show how we move from the static model, to the static intertemporal models used in productivity analysis, to the models which allow reallocation over time.

2

STATIC PRODUCTION STRUCTURE

In this chapter we introduce the basic building blocks which we will be using to develop our dynamic models of production. As in our previous work on production and performance measurement, we follow Shephard and begin by specifying axioms which technology should satisfy. These are chosen to provide the minimum structure and regularity necessary to allow us to employ duality results, for example. A brief overview of these axioms, including alternative specifications of returns to scale and disposability of inputs and outputs opens this chapter.

Having established the underlying structure of technology, we continue with a brief discussion of some behavioral models related to production. The simplest models relate directly to technology, requiring no information on prices; these include the Shephard output and input distance functions which are reciprocals of Farrell technical efficiency measures. With the introduction of prices, optimization with respect to cost, revenue or profit becomes possible, yielding dual representations of technology.

Again, as in our previous books on production and performance, our main computational tool remains the activity analysis model. Its advantages include simplicity, flexibility, computational ease and consistency with our axiom system which we demonstrate in Section 2.2. This is the same type of model used in the operations re-