FATIGUE TESTING AND ANALYSIS OF RESULTS

by

W. WEIBULL

BOCKAMÖLLAN
BRÖSARPS STATION
SWEDEN

Published for and on behalf of
ADVISORY GROUP FOR
AERONAUTICAL RESEARCH AND DEVELOPMENT
NORTH ATLANTIC TREATY ORGANIZATION

by

PERGAMON PRESS

OXFORD · LONDON · NEW YORK · PARIS
1961

PERGAMON PRESS LTD.
Headington Hill Hall, Oxford
4 & 5 Fitzroy Square, London W.1

PERGAMON PRESS INC.
122 East 55th Street, New York 22, N.Y.
Statler Center 640, 900 Wilshire Boulevard
Los Angeles 17, California

PERGAMON PRESS S.A.R.L.
24 Rue des Écoles, Paris Ve

PERGAMON PRESS G.m.b.H.
Kaiserstrasse 75, Frankfurt am Main

Library of Congress Card No. 59–14498

Set in Baskerville 10 on 11 pt. and printed in Northern Ireland at
THE UNIVERSITIES PRESS, BELFAST

TO

DERYCK C. SMITH
1916 – 1959

Executive – Structures and Materials Panel
Advisory Group for Aeronautical Research and Development
North Atlantic Treaty Organization

FOREWORD

In dedicating this volume to Deryck C. Smith, the Advisory Group for Aeronautical Research and Development wishes to commemorate the services of an outstanding member of its staff.

Mr. Smith was called to the organization to formulate a new section within the framework of AGARD. By his original ideas, his forceful personality, and his untiring devotion, he brought together a dynamic group of members for his Panel, and imbued them with his own enthusiasm for the work to be accomplished.

This volume is but one of the several publications which indicate the importance and scope of the work which was undertaken by the Panel under his guidance.

Officially AGARD has suffered a severe loss in the death of an executive who had the vision and the ability to see and to carry out an ever expanding program to increase the value of AGARD to the NATO nations.

Personally, the staff will long remember a congenial associate, a helpful and stimulating co-worker, a cherished friend.

THEODORE VON KÁRMÁN
Chairman—AGARD

CHAPTER I

SYMBOLS AND NOMENCLATURE

SECTION 10. GENERAL

There is a wide variety of symbols and nomenclature used in different countries, not to say within each country, and with few exceptions no internationally accepted standards exist. The choice of symbols to be used in the present book was not, therefore, easily taken and a definite and unobjectionable list cannot, for the time being, be established.

Under these circumstances, it was decided to follow mainly the nomenclature and symbols—some of them tentative—proposed by the ASTM Committee E–9 on Fatigue, although some modifications, chosen from the references listed below or obtained as a result of personal discussions with several experts, have been introduced.

There is one question which seems to deserve particular mention, and that is the ambiguous significance of the symbol for "stress", S, and its various subscripts. In fact, there are two quite different concepts of "stress" which are both denoted by S and which have to be kept strictly apart in order to avoid confusion. One of them is "the stress applied to the test piece", resulting from the given load; the other is "the stress at which something happens to an individual test piece", i.e. a strength value.

Into the first category fall the quantities mentioned in Section 11 such as $S_{\max}$, S_a, S_m, K_t, etc. which are factors defining the test conditions and having a magnitude which can be specified by a definite number, for example, an applied stress amplitude $S_a = 10$ kg/mm². Into the second category fall the quantities mentioned in Section 12 such as S_u, S_N, S_e, K_f, etc. which indicate some property of the material and accordingly take a value varying from specimen to specimen; in other words these quantities are random variables with a magnitude which cannot be specified by a definite number but require for their definition a distribution function or, less completely, one or more statistics; for example, the fatigue strength S_N at a given fatigue life, say $N = 10^7$, which may be specified by its arithmetic mean $\bar{S}_N$ or median $\check{S}_N$ and its lower bound S_{N_0} or variance $\sigma_S{}^2$ as a substitute for the distribution function.

Strictly speaking, quantities of the first category are non-random variables only in so far as the nominal stress applied—i.e. the stress aimed at—is concerned, which differs from the stress actually applied because of systematic or accidental errors in the calibration of the testing machine or variations in the dimensions and shape of the test piece.

The stress actually applied is evidently a random variable and thus of a character quite different from the nominal stress. Its scatter adds to the scatter due to the material. In most cases the actual stresses are unknown and only the nominal stresses are given. Consequently, no distinction between the two sources of scatter can be made and the total scatter is frequently attributed to the test piece alone. It is obvious that in cases where such a distinction is required, different symbols for nominal and actual stresses must be introduced.

REFERENCES

International Unions:

(1) International Union of Pure and Applied Physics (1955), "Symbols and Units", Document U.I.P.6, Report published with the financial support of the UNESCO.

France:

(1) Société Francaise de Metallurgie (1957), "Terminologie proposée pour la désignation des expérimentations sur la fatigue et des phénomènes liés à la fatigue", Groupe IV—Guide de la Fatigue, Document GF 3.

Germany:

(1) Deutscher Normenausschuss (1953), "Dauerschwingversuch: Begriffe—Zeichen —Durchfürung—Auswertung", Deutsche Normen, DIN 50 100.

(2) ——— (1954), "Dauerschwingversuch: Stichwortverzeichnis zu DIN 50 100 in 4 Sprachen", Deutsche Normen, DIN 50 100, Beiblatt (Vornorm).

Italy:

(1) Unificazione Italiana (1957), "Prove dei materiali metallici. Prove di fatica a temperatura ambiente: Generalita—Simboli—Definizioni", UNI 3964.

(2) Locati, L. (1942), "Terminologia nella scienza della "fatica" dei metalli", *Metallurgo Italo*, June 1942, pp. 237-241.

Netherlands:

(1) Nationaal Luchtvaartlaboratorium, Amsterdam (1954), "A proposal for fatigue symbols and nomenclature to be used in reports in the English language".

Sweden:

(1) Tekniska Nomenklaturcentralen (1946), "Benämningar och beteckningar inom hållfasthetsläran". Publ. TNC 8.

(2) Statistiska Föreningen, Stockholm (1954), "Nordisk Statistisk Nomenklatur". Engelsk-Nordisk och Svensk-Engelsk Ordlista.

United Kingdom:

(1) Royal Aeronautical Society (1958), "Terms and Notation for Aircraft Structural Fatigue". Fatigue Data Sheet G. 00.02.

United States:

(1) American Standards Association (1942), "The American Standard Letter Symbols for Concepts in Mechanics of Solid Bodies", ASA No. Z 10.

(2) American Society for Testing Materials (1937), "Nomenclature for various ranges of stress in fatigue". *Proc. Amer. Soc. Test. Mat.* Vol. 37, pp. 159-163.

(3) ——— (1948), "Symbols and Nomenclature for fatigue testing". Bull. No. 153, pp. 36-37.

(4) ——— (1949), "Symbols and Nomenclatures for fatigue testing". Section II of "Manual on fatigue testing". *Amer. Soc. Test. Mat.* STP No. 91, pp. 3-5.

(5) ——— (1955), "ASTM Standards on Plastics. Specifications—Methods of testing—Nomenclature—Definitions".

SECTION 11. APPLIED STRESS CYCLES

Stress Cycle. A stress cycle is the smallest section of the stress-time function which is repeated periodically and identically as shown in Figs. 11.1, 11.2 and 11.3. The stress cycle is defined by: (a) the stress components, (b) the shape and (c) the frequency, i.e. the number of cycles per minute or per second. The simplest shape of the cycle is the harmonic wave in which the profile is a sine or cosine curve (Fig. 11.1). The varying stress of this cycle has one maximum and one minimum value. Its damaging effect is defined by one pair of stress components. This appears to be the case also when

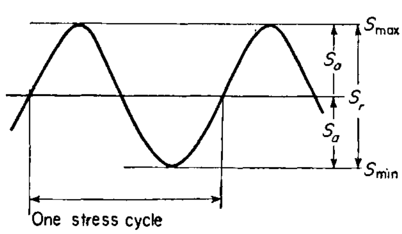

Fig. 11.1.

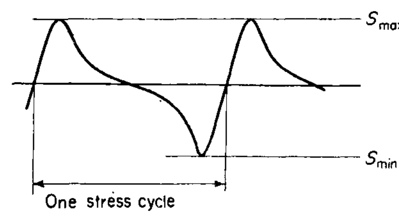

Fig. 11.2.

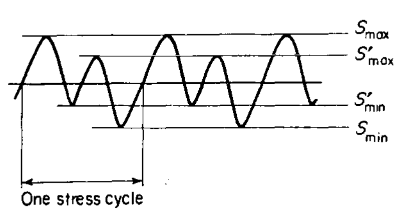

Fig. 11.3.

the wave is non-harmonic with one maximum and one minimum value as demonstrated in Fig. 11.2. A stress varying according to Fig. 11.3 requires two pairs of stress components for its definition.

Stress Level. The pair—or pairs—of stress or strain components necessary to define the applied cycle.

S = Nominal Stress. The applied stress calculated on the area of the net section of the test piece by simple theory ignoring stress raisers and disregarding plastic flow. In most of the definitions given below the word "stress" may be replaced by "load".

S_{max} = Maximum Stress. The highest algebraic value of the stress in the stress cycle, tensile stress being considered positive and compressive stress negative.

2

S_{min} = Minimum Stress.	The lowest algebraic value of the stress in the stress cycle, tensile stress being considered positive and compressive stress negative.
S_r = Range of Stress.	The algebraic difference between the maximum and the minimum stress in one cycle: $S_r = S_{max} - S_{min}$.
S_a = Stress Amplitude.	One half the range of stress: $S_a = \frac{1}{2}S_r$.
S_m = Mean Stress.	The algebraic mean of the maximum and the minimum stress in one cycle: $S_m = \frac{1}{2}(S_{max} + S_{min})$.
R = Stress Ratio.	The algebraic ratio of the minimum stress to the maximum stress in one cycle: $R = S_{min}/S_{max}$.
A = Stress Amplitude Ratio.	The ratio of the stress amplitude to the mean stress: $A = S_a/S_m$. This ratio is particularly used in high-temperature work.
K_t = Stress Concentration Factor.	The ratio of the greatest stress in the region of a notch or other stress raiser as determined by advanced theory, photoelasticity, or direct measurement of elastic strain, to the corresponding nominal stress.

SECTION 12. STRENGTHS AND FATIGUE LIMITS

S_t = Static Tensile Strength.	
S_c = Static Compressive Strength.	
S_N = Fatigue Strength.	The stress which produces fatigue failure at a number of stress cycles equal to N. The stress has to be expressed in terms of a pair of stress components, such as the stress amplitude and the mean stress, or as the maximum and the minimum stresses. One of the components is kept constant during the test, for example the mean stress, which is then a characteristic of the test conditions, while the other component, for example the stress amplitude, is a property of the material and accordingly a random variable defined by a statistical distribution function.
S_e = Fatigue Limit.	The fatigue strength for $N \rightarrow \infty$.
S_u = Ultimate Fatigue Strength.	The fatigue strength for $N \rightarrow 0$. This value is not necessarily equal to S_t or S_c.
K_f = Fatigue Notch Factor.	The ratio of the fatigue strength of a member or specimen with no stress concentration to the fatigue strength of a specimen with stress concentration.
q = Notch Sensitivity.	A measure of the degree of agreement between K_f and K_t for a particular specimen or member of given size and shape. Thus $q = (K_f - 1)/(K_t - 1)$. Notch sensitivity varies between zero (when $K_f = 1$) and unity (when $K_f = K_t$).

SECTION 13. FATIGUE LIFE AND NUMBERS OF CYCLES

N = Fatigue Life.	The number of stress cycles at which fatigue failure occurs for a given test condition.
N_r = Run-out Number (of cycles).	Number of cycles at which test is discontinued.
n = Stress Cycles Imposed.	The number of cycles which has been imposed on a specimen without failure at any stage of a fatigue test.
C = Cycle Ratio.	The ratio of the stress cycles actually applied at a given stress level to the expected fatigue life at that stress level, based on the S–N diagram: $C = n/N$.
$X = \log N$.	In some cases an unspecified random variable.
D = Fatigue Damage.	Change of fatigue properties of a test piece subjected to cycling stresses.

SECTION 14. STATISTICAL QUANTITIES AND MATHEMATICAL SIGNS

P = Probability of Failure.	The ratio of the number of specimens which have failed to the total number of specimens tested.
Q = Probability of Survival.	It follows that $P + Q = 1$.
$F(x)$ = Distribution Function of x.	A non-decreasing point function which corresponds to the probability function in such a way that $F(x) = P(\xi \leqq x)$ = the probability that the random variable ξ takes a value equal to or less than x.

$G(u)$ = Inverse Function of $F(x)$, i.e. $G[F(x)] = x$.
$f(x)$ = Frequency or Density Function of x, i.e. $dF(x)/dx = f(x)$.
$E(x)$ = Mathematical Expectation or Mean Value of a random variable ξ.
$\sigma_x^2 = D^2(x) = \text{var}\,(x)$ = Variance of x.
σ_x = Standard Deviation of x.
$\hat{\sigma}$ = Estimate of σ from a sample.
cov (x, y) = Covariance of x and y.
n and j = Sample Size = Number of values in a sample.
m and i = Order Numbers in a random sample ordered from least to greatest.
a, b and B = Parameters of an S–N equations.
α and β = Parameters of a distribution function.
$\hat{\alpha}$ and $\hat{\beta}$ = Estimates of α and β from a sample.
$\bar{X}$ = Arithmetic Mean of observed values X_m.
$\check{X}$ = Median of observed values X_m.
Σ = Summation sign.
e = Subscript corresponding to $N = \infty$.
o = Subscript corresponding to lower bound of a random variable, i.e. to $P = 0$.
$s = S - \bar{S}$ = Deviation of S from mean.
$u = U - \bar{U}$ = Deviation of U from mean.

SECTION 15. TYPES OF APPLIED LOAD CYCLE

Axial Loads

Fluctuating Tensile Load.	Minimum load and maximum load both tensile.
Repeated Tensile Load.	Minimum load zero, maximum load tensile. ($R = 0$)
Alternating Axial Load.	Unspecified axial load cycle.
Reversed Axial Load.	Alternating load with maximum load numerically equal to minimum load. ($S_m = 0$).
Repeated Compressive Load.	Maximum load zero, minimum load compressive.
Fluctuating Compressive Load.	Minimum load and maximum load both compressive.

Plain Bending Loads

Fluctuating, repeated, alternating and reversed bending loads defined analogically with definitions for axial loads.

Rotating Bending Loads

A rotating specimen is subjected to a constant non-rotating bending moment, or a non-rotating specimen is subjected to a rotating constant bending moment.

Torsional Loads

Fluctuating, repeated, alternating and reversed torsional loads defined analogically with definitions for axial loads.

Combined Loads

To be specified for each condition, including any relative phase differences between the components.

SECTION 16. VARIABLE-STRESS LEVEL TESTS

Variable-stress Level Test.	Test during which a specimen is subjected to stress cycles differing in stress amplitude and/or mean stress.
Step.	Fixed number of stress cycles of constant amplitude and mean stress.
Block.	An aggregate of steps.
Shape of Block.	The pattern in which the steps are arranged within the block.
Size of Block.	Total number of cycles or value of $\Sigma n/N$ of the block or estimated number of blocks to failure.
Period.	Fixed number of stress cycles of magnitude varying continuously according to a given pattern.
Preload Test.	A fatigue test which is preceded by a number of high loads.
Prestress.	A step preceding the last stress level which is continued until failure occurs.
Programme Test.	Load is composed of a limited number of steps, usually grouped into blocks which are repeated until failure occurs.
Randomized Programme Test.	The sequence of the steps is random.
Spectrum Test.	Consecutive stress cycles are of different magnitude.

CHAPTER II

FATIGUE TESTING METHODS

SECTION 20. GENERAL

The objective of a fatigue test is, generally speaking, to determine the fatigue life and/or the danger point, i.e. the location of failure, of a test piece subjected to a prescribed sequence of stress amplitudes. In some specific cases this may be the sole purpose of the test; e.g. if the test piece is a complicated machine part or an assembly of components and the applied load is a sequence of varying stress amplitudes intended to simulate the stress history encountered in actual service.

In most cases, however, it is required that the test be designed in such a way that it does not only answer the specific question which has been put, but will also allow a generalization of the result obtained and contribute to the discovery of laws or rules relating fatigue life with various influential factors. For this purpose it is indispensable that the test conditions be simplified, be it with regard to the sequence of stress amplitudes or to the test piece or to both of these factors. By simplifying and idealizing the test conditions it will be possible to vary one or a few of the factors which influence the fatigue life and to state their effects.

Even if these conditions are fulfilled, there will always remain a number of unknown and uncontrollable factors which produce a large scatter in fatigue life even of test pieces which are considered to be identical. In the past, this scatter in fatigue life was not regarded as a problem and only a few specimens were used to determine the fatigue limit or the relation between load and life. It is now generally accepted that the scatter is an inherent part of the fatigue properties, and that a large number of specimens is required even if average values only are concerned. This requirement has some influence on the choice of the testing procedure.

The two above-mentioned factors: (i) the sequence of stress amplitudes and (ii) the test piece, will now be used as a basis for a classification of the different methods of fatigue testing.

The simplest sequence of amplitudes is obtained by applying reversals of stress of a constant amplitude to the test piece until failure occurs. Different specimens of the test series may be subjected to different stress amplitudes, but for each individual item the amplitude will never be varied. This type of fatigue testing is called a *constant-amplitude test.*

Depending upon the choice of stress levels, constant-amplitude tests may be classified into three categories:

(i) *the routine test,* where applied stresses are chosen in such a way that all specimens are expected to fail after a moderate number of cycles, say 10^4 to 10^7. A few run-outs, although not intended, may be allowed;

(ii) *the short-life test*, where stress levels are situated above the yield stress and some of the specimens are expected to fail statically at the application of the load; and

(iii) *the long-life test*, where stress levels are situated below or just above the fatigue limit and a fraction of the specimens does not fail after a preassigned number of cycles (usually between 10^6 and 10^7).

Obviously, there is no abrupt transition from one type to another. Suppose for example that five samples of equal size n are drawn at random from a real or hypothetical population and tested at five different stress levels as indicated in Fig. 20.1; then it may be postulated that all specimens having the same

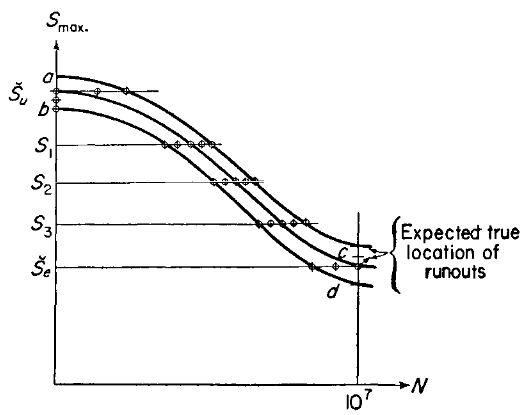

Fig. 20.1. *P–S–N* diagram including statically fractured specimens and run-outs.

order number, from least to greatest fatigue life, will have approximately identical static and fatigue properties—the larger the sample, the smaller the individual deviations from average—which are represented by a family of *S–N* curves, of which three, the median and the two extreme curves, are indicated in the Fig. The estimated range of the tensile strength S_u for the given sample size is marked by *a–b*, and that of the fatigue strength S_N ($N = 10^7$, say) is marked by *c–d*. If now the stress level $S = \check{S}_u$, fifty per cent of the specimens tested are expected to fail within the first cycle ($N = 0$) whereas the remaining half are expected to have a fatigue life $N \geqq 1$. In the same way, if $S = \check{S}_N$, fifty per cent of the specimens tested are expected to endure more than 10^7 stress reversals.

A short-life test thus includes stress levels above the lower bound of the tensile strength (point *b*) and a long-life test includes stress levels below the upper bound of the fatigue strength (point *c*).

In some cases it will be required to substitute more complicated sequences of stresss amplitudes than constant ones. The sequence obtained by subjecting each test piece to reversals of monotonic increasing amplitude is called *the increasing-amplitude test*. It is a typical long-life test, exclusively used for the same purpose as the response test (see Section 23, paragraph 1), and it is, therefore, in spite of not being a constant-amplitude test, presented in the same section as the response test. The increase in amplitude may be either by steps or continuous as demonstrated in Section 23, paragraph 2.

More complicated sequences of amplitude are required in order to simulate the stresses to which a specimen is subjected in actual service. A realistic simulation is very complicated. In order to discover laws in relation to the accumulation of fatigue damage in a specimen subjected to stress reversals of different amplitudes, the sequence of stress amplitudes, also called the programme or the spectrum of loading, may be simplified. Independent of the pattern used such tests will be designated *variable-amplitude tests*, the only exception being the monotonic increasing-amplitude test which is regarded as a category by itself. Two alternatives will be considered. If the objective of the test is to investigate cumulative damage theory, in which case the sequence is frequently simplified, composed of perhaps two or three stress levels only, the test will be called the *cumulative-damage test*, discussed in Section 24, whereas tests using a more elaborate pattern for simulating purposes will be called the *service-simulating tests*, discussed in Section 25.

Having thus classified the various fatigue tests on the basis of the sequence of stress amplitudes, subclasses may be obtained by considering the different *types of test piece* available. It will suffice to divide the test pieces into two categories, which will be designated *specimens* and *components*.

The term *specimen* is here used in the sense of a test piece of simple shape, frequently standardized, of small size, and prepared carefully and with good surface finish. The purpose of the simplification is not only to make it less expensive but more to reduce the variability of the product and to keep different influential factors under control. Test pieces of this type were originally intended for testing the material and for stating its fatigue properties. They are now also used extensively for research purposes.

Even if the simplified specimen may simulate many of the properties of actual machine parts, there are two factors pertaining to the component which are not represented in the specimen, i.e. design and fabrication. For this reason it is indispensable to carry out actual tests with components in exactly the same condition as used in actual service.

The term *component* is here used to signify any machine part, actual structure, machine and assembly, including elements simulating actual components.

The different types of test mentioned above may be applied either to specimens or to components. Of the different combinations possible, TEMPLIN (1949) has paid particular attention to three of these combinations, viz. the routine test applied to specimens and components and the service-simulating test applied to components. They have been designated by him as the *material test*, the *structural test* and the *actual-service test*.

It may be appropriate to mention some of the purposes for which data from such tests are intended.

Tests of the *material type* are useful for a comparison of the behaviour of different materials subjected to repeated stresses, of the effects of various manufacturing processes, of the behaviour of materials in various environments, of various simple geometrical factors such as different sizes and shapes of notches, and different surface finishes. They may also be used to establish correlations with other mechanical properties, different types of stressing, chemical compositions and for evaluating the effects of surface treatments such as case-hardening, decarburization, nitriding, shot-peening and plating on the fatigue properties of different materials.

Tests of the *structural type* may be useful for a comparison of components made from different materials, of different design and of structures fabricated by different procedures. They may also be used for revealing stress concentrations and fabrication faults, for developing better designs or fabrication procedures and for establishing design criteria. In some cases, the location of this failure point is the only information required (De Leires, 1956).

All fatigue tests are very time-absorbing, particularly when a number of tests sufficiently large to allow statistical treatment is required. This difficulty has been apparent to research workers almost from the beginning of fatigue testing, and several methods have been suggested in an attempt to discover some rapid method which could be substituted for the normal fatigue testing methods. Such abbreviated and accelerated tests are discussed in Section 26.

Fatigue tests completely different in type from the above-mentioned tests are those which have as objective a study of the initiation and propagation of fatigue cracks. In the routine tests the most common practice is to run the test until complete fracture of the specimen occurs. From a theoretical point of view, it would be much better to split up the test into two parts. The pre-crack stage and the post-crack stage, owing to the fact that the fatigue damage is of a quite different character in these two stages. Simple laws are therefore not to be expected without such a separation. This is perhaps particularly true when size effects and similar problems are concerned. Some comments on tests intended for the determination of the crack initiation and for a study of the crack propagation are to be found in Section 27.

The above-mentioned methods must be modified for *certain special purposes*. Some particular cases are indicated and references are given in Section 28.

References: Belyaev (1951), Berg (1941), Cazaud (1934), Christol (1937), De Leires (1956), Franke (1929), Gillett, Grover and Jackson (1946), Gough and Clenshaw (1935), Johnstone (1947), Moore (1925), Moore, Spargen and Claussen (1938), Peterson (1945), Siebel (1938), Siebel and Ludwig (1953–1957), Sigwart and Petersen (1953), Templin (1948).—ASTM STP91 (1949), ASTM STP91—A (1958), DIN 50100 (1953).

SECTION 21. ROUTINE TESTS

The purpose of the routine test is to estimate the relation between load and life; in the past, with the chief aim of determining the fatigue limit by an extrapolation of the curve fitted by eye to the data points.

Later it has become apparent that not too much confidence should be placed on results obtained from an extrapolation of empirical curves carried out without using proper caution, and since more powerful tests for stating long-life fatigue properties have been available, the use of a routine test should be restricted to the range of stress levels actually studied. (The problem of extrapolating curves to ranges outside the observations is discussed in Sections 71 and 91.)

This type of test is usually designed with the intention of having all the specimens fail. There is, however, in some cases and for some purposes reason to discontinue the test when a certain fraction at each stress level has failed, and the routine tests may then be classified into *all-failed* and *fraction-failed tests.*

21.1 All-failed Tests

The purpose of the all-failed test is usually to determine the relation between the fatigue life and the amplitude of the applied stress for the test piece used, keeping the mean stress S_m or the stress ratio R constant. The result and its usefulness depend upon the total number of specimens, the choice of stress levels, and the allocation of specimens to the stress levels.

If the total number of specimens is small, the only information obtainable is an estimate of the average S–N curve corresponding to a probability of failure (or of survival) of about fifty per cent. In the past, before designing for limited life was actually needed, this was all that was required of the test. It was considered neither necessary nor desirable to use many specimens for each test series. The normal procedure was to run a single test at each stress level, reducing the range of stress with each succeeding specimen. The pretensions were very moderate indeed. It was stated that the determination of the limiting stress of a metal could be determined with "a number of specimens which cannot be safely reduced below four, even under the best circumstances".

Findley (1949) suggests that at least ten specimens be tested for an S–N diagram, but that a larger number of specimens would be desirable for establishing the S–N diagram accurately and indicating the variability of the material. He proposes that for this purpose at least 20 (preferably 50) specimens should be prepared and tested.

It has been experimentally verified (Weibull, 1958a) that, even if the number of specimens tested has a self-evident influence on the accuracy of the parameters computed from the observations, other factors may be of equal importance. In some cases, small test series could give just as good or even better accuracy than series three or four times as large. The efficiency of a test series in this respect depends also upon the choice of the stress levels, the inherent scatter of the specimens used and of the testing machine and possibly of some other factors; so, in a way, a small number of specimens can to some extent be compensated by a more efficient design of the test conditions. This problem will, however, be more thoroughly discussed in Section 71.

It is believed that some twenty to thirty specimens will give a fair estimate of the variance of the fatigue strength and that fifty to one-hundred specimens will be required for establishing an acceptable P–S–N diagram, provided efficient statistical methods are used for the evaluation of the observed data.

The *choice of stress levels* depends upon the purpose for which the data are required. If the main interest is in the long-life range of the S–N curve, low stress levels will be chosen. If the complete S–N diagram or the P–S–N diagram is wanted, the stress levels may be more evenly distributed. It is strongly recommended that some static tests should also be included, if possible using specimens identical to those used in the fatigue tests. It is desirable to introduce the experimentally determined value of the static tensile strengths S_t as a unit and to use relative stresses, i.e. to express the stresses as percentage of S_t, because parameters referring to relative stresses have a more general validity than if the stresses are given in absolute dimensions.

The influence of the magnitude of the stress levels on the efficiency of the test series with regard to the accuracy of computed parameters may briefly be stated by saying that the greater the difference between the highest and the lowest stress levels, the greater the accuracy. Also from this point of view it is advantageous if the static strength S_t can be used as an integrating part for the evaluation of the test data.

The *allocation of tests to the stress levels* is not very crucial on condition that a proper transformation of the quantities (S, N) has been performed, resulting in a homogeneous variance of the variables, as demonstrated in Section 91. All the observations can then be pooled and used to determine the distribution of the deviations from the average curve. Frequently, the best method appears to be to allocate an equal number of tests to the stress levels; the fitting of P–S–N diagrams can then be performed more easily as demonstrated in Section 94.

Since the numbers of specimens at each stress level have been decided, attention must be paid to an unbiased distribution of the items. The problem of designing the test series properly is discussed more thoroughly in Section 71.

References: Findley (1949), Findley, Century and Hendrickson (1952), Müller (1937), Weck (1950), Weibull (1958a), Wellinger (1955), von Zeerleder (1935)—DIN 50142 (1941), DIN 50113 (1952), DVM Specifications (1933), French Air Ministry (1938).

21.2 Fraction-failed Tests

For practical design purposes it is of little interest to know the fatigue life of the better specimens of a fatigue tested group, as the designer has to base his calculations on the worst part of the group. It would be quite sufficient for him to have a safe knowledge of the lower part of the life or strength distribution.

Since the total time required for a test series is largely determined by the long-life items, it is obvious that a considerable saving in time may be obtained by stopping the tests when a certain fraction of the group has failed. For example, a series of 120 specimens allocated to five stress levels (Weibull, 1956c, Table 1) required a total machine time of 144·2 million cycles, the 12 smallest values of each stress level taking 17·3 million or 12 per cent and the 12 largest taking 126·8 million or 88 per cent of the total time. If the latter had been stopped at the median values of life, a saving of

91·8 million cycles would have resulted. The total time of the 50 per cent fraction-failed series is thus 36·3 per cent of that of the all-failed series.

Still more reduction in testing time will result according to a "least-of-four method", proposed by SCHUETTE (1954). Four specimens are tested simultaneously and the test is discontinued as soon as one of them has failed. By means of these data an *S–N* curve for approximately 80 per cent survival is obtained.

If the observations are evaluated by efficient statistical methods not very much design information is lost by testing a fraction only. Such methods are discussed in Sections 91–94. A reduction of the time required for the experiment can be important when the results are needed as soon as possible or when the cost associated with a failed item is much larger than the cost of a life-tested item which did not fail.

There is no fundamental difference in testing technique between this type and the all-failed test. If a sufficient number of testing machines is available for simultaneous testing, the test can be stopped at exactly the desired fraction. Otherwise a safe value of the median life for each stress level must be estimated and an approximate fraction of failures will result.

This type of test may be regarded as a modification of the all-failed test and it is run for the same purpose, i.e. to establish the *S–N* diagram or part of the *P–S–N* diagram. The alternative fraction-failure test, the response test, where the tests are stopped at a preassigned cycle life, equal for all stress levels, is different in character and has another objective. It will therefore be discussed in a separate Section.

References: SCHUETTE (1954), WEIBULL (1955a, 1956c).

SECTION 22. SHORT-LIFE TESTS

By far the greater part of conventional fatigue testing has been concerned with establishing fatigue lives at stresses well below the yield stress of the material. In some cases, however, optimum design requires knowledge of the behaviour of the material under stresses leading to fatigue failure after a small number of stress—or strain—reversals.

One of the difficulties associated with testing at stresses producing large plastic deformations is the accurate control of applied loads, in particular of the mean stress. For this reason, it appears easier to base the testing equipment on the strain amplitude, rather than on the stress amplitude. It must be emphasized, however, that there is a basic difference between curves relating stress and fatigue life and curves relating strain and fatigue life, and at present it is impossible to transform one to the other.

It is obvious that these two modes of stressing are equivalent as long as the test piece is acting as a perfect elastic body, i.e. as long as there is a unique relation between displacement and applied load. This condition may, at low stresses, be fulfilled during the first stage of the fatigue life, but it will be invalidated as soon as cracks appear. At high stresses, it may be invalidated even during the first stress reversals. As an example reference is made to a paper by LIU *et al.* (1948). Unnotched specimens of aluminium alloy 24S–T were subjected to completely reversed axial strains of such a magnitude that failure occurred in some seven cycles. The maximum true stress in each succeeding cycle increased until it had reached a value of 12 per cent higher than the initial value.

Another example is reported by Low (1956). A preset angular movement was applied to the ends of a flat rectangular test piece. The curvature at the test section, and therefore the maximum fibre strain, amounting to a value of up to 5 per cent, was determined by a spherometer. Preliminary tests showed that the spherometer readings remained the same throughout the greater part of a test, but once localized yielding or cracking of the test piece occurred, the angular movement, required to give the same reading, altered considerably. It is obvious that the fatigue life observed will depend considerably on whether the preset angular movement of the testing machine is changed or not. A proper interpretation of the result of a short-life test thus requires a more detailed description of the test conditions.

Usually different testing machines have to be used to cover the complete range of the *S–N* curves. Tests in which failure occurs in less than 10 kc are impracticable to perform with most of the conventional testing machines. Tests in which failure is expected to occur in 0·5 to 10 kc are frequently carried out with hydraulically operated testing machines, whereas failures expected to appear in less than 500 cycles are usually performed by the use of manually operated machines. For this purpose, conventional static testing machines may be used. The speed is, of course, very low. A few cycles per minute may be obtained in this way. A reduction of the speed is required not only because of the machine but in order to keep the heating of the test piece, due to large plastic deformations, within reasonable limits.

For all specimens tested at stress levels higher than the yield strength of the material, it is advisable to apply the first reversal of load manually in order to produce the plastic deformation. This procedure simplifies the maintenance of the desired mean load.

From the preceding, it is apparent that short-life tests have to be divided into constant-stress amplitude and constant-strain amplitude tests.

Methods of analysing data from fatigue tests including static fractures are discussed in Section 91.

22.1 Constant-stress Amplitude Tests

Available data on fatigue testing of steel specimens at stresses producing failure in less than 30 kc are summarized by Weisman and Kaplan (1950). Only a few of the data are for tests resulting in failure in less than 1 kc. They were performed on unnotched specimens subjected to bending and to axial load at a stress ratio $R = 0$.

Tests with notched specimens of steel and of 61S–T6, 24S–T3 and 75S–T6 aluminium alloys have been conducted by Hardrath and Illg (1954). A most remarkable result was that the minimum life to failure at stresses near the ultimate strength was drastically reduced with increasing stress-concentration factor. Failure was found to occur in approximately 10 kc for unnotched specimens, 1 kc for specimens with $K_t = 2$, and in 0·1 kc for specimens with $K_t = 4$. Further, in tests with $R = -1$ and $K_t = 4$, the

S against log N curves were found to be concave upwards for almost the complete range, a reversal in curvature occurring at about 10 cycles of reversals.

References: HARDRATH, LANDERS and UTLEY (1953), HARDRATH and ILLG (1954), WEISMAN and KAPLAN (1950).

22.2 Constant-strain Amplitude Tests

Tests of this type were already in use by KOMMERS (1912) who applied maximum fibre strains in the range of 2·5 to 0·7 per cent to specimens of steel. A bending fatigue test including five widely differing materials, steels and aluminium alloys, is reported by LOW (1956). The fatigue life in reversed bending was found to depend solely on the degree of strain, and is independent of the material for maximum fibre strains between ± 5 and ± 4 per cent. In tests using lower strains, the fatigue depended also on the material. Curves of deflexion against cycle life were found to be smooth over the whole range, from which it follows that the curves of stress against cycle life all show an abrupt change of slope at the yield stress of the material. It is a remarkable result that all the curves plotted on log–log scales are, within a reasonable, non-systematic scatter, identical. The slope $\text{d} \log N/\text{d} \log S = -2{\cdot}4$ (S denoting the strain). This result agrees very closely with that obtained by KOMMERS (1912).

Tests of this type are described also by LIU *et al.* (1948) as mentioned above and by PARDUE *et al.* (1950). The latter investigation examines specimens of seven different materials subjected to strain reversals resulting in failure in less than 10 kc.

References: KOMMERS (1912), LIU, LYNCH, RIPLING and SACHS (1948), LOW (1956), PARDUE, MELCHOR and GOOD (1950).

SECTION 23. LONG-LIFE TESTS

The object of the long-life test is to determine a number of percentage points of the distribution of the fatigue strength at a preassigned cycle life. It differs from the routine test in that the observed values of fatigue life are not used directly, only the fraction that failed at different stress levels being used. This procedure obviously means a loss of some of the information which is provided by the test. It is therefore recommended that the observed cycles-to-failure should be regarded as part of a routine test, and used accordingly.

The long-life tests may be classified into a constant-amplitude test, which is called *the response test*, and *the increasing-amplitude test.*

23.1 Response Tests

The response test is conducted according to two different methods. The first, using *the probit method,* is designed with predetermined stress levels and numbers of specimens at each stress level; the second, using *the stair-case method,* is a sequential test, the choice of stress level is determined by the preceding result.

23.11 ***The probit method.***—The object of the probit method is to determine the complete distribution function of the fatigue strength or part of it. The examination may be concentrated to different parts of the distribution, but the number of tests required for a safe estimate of extreme percentage points would be prohibitive.

The common procedure is to divide the specimens available into several groups and to test one group at a chosen stress level, the next group at a second level, and so on. The data which are used for the evaluation consist of the numbers of failures and non-failures at each stress level.

The stress levels are chosen in such a way that one of them will give a fraction of failures prior to the preassigned fatigue life estimated to be equal to the percentage of main interest, be it 50 per cent or some other value. It is recommended that there should be two stress levels above and two below the mean level. If the region of the median is of main interest the stress levels could be located close together, and sometimes three levels would be sufficient. If more general information is desired, the levels ought to be more widely spread.

The analysis of the data may be made graphically or analytically. In any case, if equal groups have been used a weighting procedure is required. This complication can be eliminated by allocating more tests to percentage points corresponding to large variance of the observations. If the distribution is assumed to be normal, the following table indicates appropriate sizes of the groups. This table may also apply to distributions other than normal.

An acceptable accuracy of the response curve, including confidence limits, will require a total number of some fifty specimens.

Methods for analysing the data are discussed in Section 95, paragraph 1.

Expected Percentage Survival	*Relative Group Size*
25 to 75	1
15 to 20	1·5
80 to 85	1·5
10 to 90	2
5 to 95	3
2 to 98	5

(From the ASTM STP 91-A)

References: BLISS (1935a,b, 1937), FINNEY (1952), FISHER and YATES (1943), GOLUB and GRUBBS (1956), MOORE and WISHART (1933).

23.12 ***The staircase method.***—If the main interest is limited to the median value of the fatigue strength the stair-case method will reduce the number of specimens required. On the other hand, it is not a good method for estimating small or large percentage points unless the distribution is assuredly normal.

The procedure of the staircase method is as follows. The first test is started at a stress level which is equal to an estimated mean value of the fatigue strength. If a failure occurs prior to the preassigned cycle life, the next specimen is tested at a lower level; if the specimen does not fail within