Advances in Fuzzy Mathematics and Engineering

Fuzzy Sets and Fuzzy Information-Granulation Theory

Key Selected Papers by Lotfi A. Zadeh

Beijing Normal University Press

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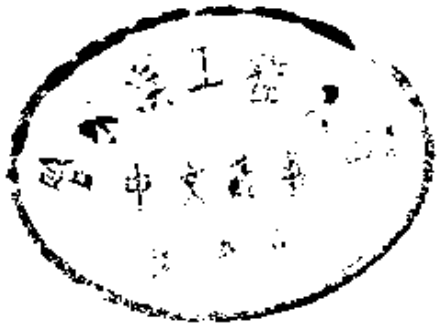
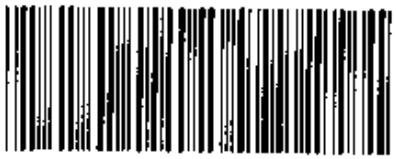
Edited by Da Ruan
Chongfu Huang





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Preface

Advances in Fuzzy Mathematics and Engineering is a new international series dedicated to the support and development of the theory of fuzzy mathematics and related areas and their industrial applications in general and in engineering in particular. The series is supported and published by Beijing Normal University Press, Beijing, China.

This book, *Fuzzy Sets and Fuzzy Information-Granulation Theory*, is the third volume of Collected Papers by Lotfi A. Zadeh. The first volume, entitled *Fuzzy Sets and Applications*, was published in 1987 by **John Wiley**. Its editors, Ronald R. Yager, Sergei Ovchinnikov, Richard M. Tong, and Hung T. Ngugen undertook the project on the occasion of the 20th anniversary of the publication of the first paper on fuzzy set by Lotfi A. Zadeh. The second volume, entitled *Fuzzy Sets, Fuzzy Logic, and Fuzzy Systems*, was published in 1996 by **World Scientific**. Its editors, George J. Klir and Bo Yuan selected, from among all papers by Lotfi Zadeh not included in the first volume, those papers on fuzzy sets, fuzzy logic, and fuzzy systems whose easy accessibility would likely be of benefit to those working in these areas. The previous two volumes in English have proved to be great utility to anyone interested in fuzzy set theory and its applications.

Considering the largest number of the readers related to Fuzzy Mathematics and Engineering in China, we felt that a Chinese version of the key selected papers by Lotfi Zadeh *Fuzzy Sets and Fuzzy Information-Granulation Theory* would fit well into the book series on **Advances in Fuzzy Mathematics and Engineering** by Beijing Normal University Press. The book collects Zadeh's original perception which may be viewed as an evolution of ideas rooted in his 1965 paper on *fuzzy sets*; 1971 paper on *fuzzy systems*; 1973 — 1976 papers on *linguistic variables*, *fuzzy if-then rules* and *fuzzy graphs*; 1979 paper on *fuzzy sets and information granularity*; 1986 paper on *generalized constrains*; 1996 paper on *computing with words and* 1997 papers on *theory of fuzzy information granulation.*

The purpose of this book is twofold. Firstly, it is intended as a quick reference for those working in *Fuzzy Mathematics and Engineering* in China as well as in the world. Secondly, it is expected to play a major role in-Research and Development of *Fuzzy Mathematics and Engineering*, as a useful source of supplementary readings in this new book series. We hope this volume will benefit many readers around the world.

Da Ruan, Chongfu Huang
Editors

Note to the Reader

This book is the third volume of collected papers by Lotfi A. Zadeh. The first volume, entitled *Fuzzy Sets and Applications*, was published in 1987 by **John Wiley** (Editors: Ronald R. Yager, Sergei Ovchinnikov, Richard M. Tong, and Hung T. Ngugen). The second volume, entitled *Fuzzy Sets, Fuzzy Logic, and Fuzzy Systems*, was published in 1996 by **World Scientific** (Editors: George J. Klir and Bo Yuan).

The current edited-book is a set of key selected papers by Lotfi Zadeh. Both English and Chinese versions of these papers are available.

Acknowledgements

This book consists of the following reprinted (both retyped and translated) papers. The relevant copyright owners whose permissions to reproduce the papers in this book are gratefully acknowledged.

Academic Press:

L. A. Zadeh, "Fuzzy sets." *Information and Control*, 8(3), pp. 338—353, 1965.

L. A. Zadeh, "A fuzzy-algorithmic approach to the definition of complex or imprecise concepts," Internat. J. Man-Machine Stud. 8, pp. 249～291, 1976.

Elsevier Science:

L. A. Zadeh, "The concept of a linguistic variable and its application to approximate reasoning, I, II, III." *Information Sciences*, 8(3), pp. 199～251, (4)301～357; 9, pp. 43～80, 1975.

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L. A. Zadeh, "Toward a theory of fuzzy information granulation and its centrality in human reasoning and fuzzy logic." *Fuzzy Sets and Systems* 90 (2), pp. 111～127, 1997.

Institute of Electrical and Electronics Engineers:

L. A. Zadeh, "Outline of a new approach to the analysis of complex

system and decision processes." *IEEE Trans. On Systems, Man, and Cybernectics*, 1(1), pp. 28～44, 1973.

L. A. Zadeh, "Fuzzy logic = computing with words." *IEEE Trans. On Fuzzy Systems* 4, pp. 103～111, 1996.

Oxford University Press Inc.:

L. A. Zadeh, "Towards a theory of fuzzy systems. "In: Kalman, R. E. and N. Declaris, eds., *Aspects of Networks and Systems* Theory. Holt, Rinehart & Winston, New pp. 469～490, 1971.

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L. A. Zadeh, "Outline of a computational approach to meaning and knowledge representation based on the concept of a generalized assignment statement. "In: Thoma, M. And W. Wyner, eds., *Proc. Of the International Seminar on AI and Man-Machine Systems*. Springer-Verlag, Heidelberg, pp. 198～211, 1986.

Lotfi A. Zadeh's biography

Lotfi A. Zadeh joined the Department of Electrical Engineering at the University of California, Berkeley, in 1959, and served as its chairman from 1963 to 1968. Earlier, he was a member of the electrical engineering faculty at Columbia University. In 1956, he was a visiting member of the Institute for Advanced Study in Princeton, New Jersey. In addition, he held a number of other visiting appointments, among them a visiting professorship in Electrical Engineering at MIT in 1962 and 1968; a visiting scientist appointment at IBM Research Laboratory, San Jose, CA, in 1968, 1973, and 1977; and visiting scholar appointments at the AI Center, SRI International, in 1981, and at the Center for the Study of Language and Information, Stanford University, in 1987—1988. Currently he is a Professor in the Graduate School, and is serving as the Director of BISC (Berkeley Initiative in Soft Computing).

Until 1965, Dr. Zadeh's work had been centered on system theory and decision analysis. Since then, his research interests have shifted to the theory of fuzzy sets and its applications to artificial intelligence, linguistics, logic, decision analysis, control theory, expert systems and neural networks. Currently, his research is focused on fuzzy logic, soft computing and computing with words.

An alumnus of the University of Teheran, MIT, and Columbia

University, Dr. Zadeh is a fellow of the IEEE, AAAS, ACM and AAAI, and a member of the National Academy of Engineering. He was the recipient of the IEEE Education Medal in 1973 and a recipient of the IEEE Centennial Medal in 1984. In 1989, Dr. Zadeh was awarded the Honda Prize by the Honda Foundation, and in 1991 received the Berkeley Citation, University of California.

In 1992, Dr. Zadeh was awarded the IEEE Richard W. Hamming Medal "For seminal contributions to information science and systems, including the conceptualization of fuzzy sets." He became a Foreign Member of the Russian Academy of Natural Sciences (Computer Sciences and Cybernetics Section) in 1992 and received the Certificate of Commendation for AI Special Contributions Award from the International Foundation for Artificial Intelligence. Also in 1992, he was awarded the Kampe de Feriet Medal and became an Honorary Member of the Austrian Society of Cybernetic Studies.

In 1993, Dr. Zadeh received the Rufus Oldenburger Medal from the American Society of Mechanical Engineers "For seminal contributions in system theory, decision analysis, and theory of fuzzy sets and its applications to AI, linguistics, logic, expert systems and neural networks." He was also awarded the Grigore Moisil Prize for Fundamental Researches, and the Premier Best Paper Award by the Second International Conference on Fuzzy Theory and Technology. In 1995, Dr. Zadeh was awarded the IEEE Medal of Honor "For pioneering development of fuzzy logic and its many diverse applications." In 1996, Dr. Zadeh was awarded the Okawa Prize "For Outstanding contribution to information science through the development of fuzzy logic and its applications."

In 1997, Dr. Zadeh was awarded the B. Bolzano Medal by the Academy of Sciences of the Czech Republic "For outstanding achievements in fuzzy mathematics." He also received the J. P. Wohl Outstanding Career

Achievement Award of the IEEE Systems, Man and Cybernetics Society. He served as a Lee Kuan Yew Distinguished Visitor, lecturing at the National University of Singapore and the Nanyang Technological University in Singapore, and as the Gulbenkian Foundation Visiting Professor at the New University of Lisbon in Portugal.

Dr. Zadeh holds honorary doctorates from Paul-Sabatier University, Toulouse, France; State University of New York, Binghamton, NY; University of Dortmund, Dortmund, Germany; University of Oviedo, Oviedo, Spain; University of Granada, Granada, Spain; Lakehead University, Canada; University of Louisville, KY; Baku State University, Azerbaijan; and the Silesian Technical University, Gliwice, Poland.

Dr. Zadeh has authored close to two hundred papers and serves on the editorial boards of over fifty journals. He is a member of the Technology Advisory Board, U. S. Postal Service; Advisory Committee, Department of Electrical and Computer Engineering, UC Santa Barbara; Advisory Board, Fuzzy Initiative, North Rhine-Westfalia, Germany; Fuzzy Logic Research Center, Texas A &. M University, College Station, Texas; Advisory Committee, Center for Education and Research in Fuzzy Systems and Artificial Intelligence, Iasi, Romania; Senior Advisory Board, International Institute for General Systems Studies; the Board of Governors, International Neural Networks Society.

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Part 5: Soft computing with words

Part 1: Fuzzy Sets

Fuzzy Sets

A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership (characteristic) function which assigns to each object a grade of membership ranging between zero and one. The notions of inclusion, union, intersection, complement, relation, convexity, etc., are extended to such sets, and various properties of these notions in the context of fuzzy sets are established. In particular, a separation theorem for convex fuzzy sets is proved without requiring that the fuzzy sets be disjoint.

1. Introduction

More often than not, the classes of objects encountered in the real physical world do not have precisely defined criteria of membership. For example, the class of animals clearly includes dogs, horses, birds, etc. as its members, and clearly excludes such objects as rocks, fluids, plants, etc. However, such objects as starfish, bacteria, etc. have an ambiguous status with respect to the class of animals. The same kind of ambiguity arises in the case of a number such as 10 in relation to the "class" of all real numbers which are much greater than 1.

Clearly, the "class of all real numbers which are much greater than 1," or "the class of beautiful women," or "the class of tall men," do not constitute classes or sets in the usual

mathematical sense of these terms. Yet, the fact remains that such imprecisely defined "classes" play an important role in human thinking, particularly in the domains of pattern recognition, communication of information, and abstraction.

The purpose of this note is explore in a preliminary way some of the basic properties and implications of a concept which may be of use in dealing with "classes" of the type cited above. The concept in question is that of a *fuzzy set*,① that is, a "class" with a continuum of grades of membership. As will be seen in the sequel, the notion of a fuzzy set provides a convenient point of departure for the construction of a conceptual framework which parallels in many respects the framework used in the case of ordinary sets, but is more general than the latter and, potentially, may prove to have a much wider scope of applicability, particularly in the fields of pattern classification and information processing. Essentially, such a framework provides a natural way of dealing with problems in which the source of imprecision is the absence of sharply defined criteria of class membership rather than the presence of random variables.

We begin the discussion of fuzzy sets with several basic definitions.

2. Definitions

Let X be a space of points (objects), with a generic element

① An application of this concept to the formulation of a class of problems in pattern classification is described in RAND Memorandum RM-4307-PR, "Abstraction and Pattern Classification," by R. Bellman, R. Kalaba and L. A. Zadeh, October, 1964.

of X denoted by x. Thus, $X=\{x\}$.

A *fuzzy set (class)* A in X is characterized by a *membership (characteristic) function* $f_A(x)$ which associates with each point[①] in X a real number in the interval $[0,1]$[②], with the value of $f_A(x)$ at x representing the "grade of membership" of x in A. Thus, the nearer the value of $f_A(x)$ to unity, the higher the grade of membership of x in A. When A is a set in the ordinary sense of the term, its membership function can take on only two values 0 and 1, with $f_A(x)=1$ or 0 according as x does or does not belong to A. Thus, in this case $f_A(x)$ reduces to the familiar characteristic function of a set A. (When there is a need to differentiate between such sets and fuzzy sets, the sets with two-valued characteristic functions will be referred to as *ordinary sets* or simply *sets*.)

Example. Let X be the real line R^1 and let A be a fuzzy set of numbers which are much greater than 1. Then, one can give a precise, albeit subjective, characterization of A by specifying $f_A(x)$ as a function on R^1. Representative values of such a function might be: $f_A(0)=0$; $f_A(1)=0$; $f_A(5)=0.01$; $f_A(10)=0.2$; $f_A(100)=0.95$; $f_A(500)=1$.

It should be noted that, although the membership function of a fuzzy set has some resemblance to a probability function

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① More generally, the domain of definition of $f_A(x)$ may be restricted to a subset of X.

② In a more general setting, the range of the membership function can be taken to be a suitable partially ordered set P. For our purposes, it is convenient and sufficient to restrict the range of f to the unit interval. If the values of $f_A(x)$ are interpreted as truth values, the latter case corresponds to a multivalued logic with a continuum of truth values in the interval $[0,1]$.