

# **ELECTRONIC COUNTERS**

**R.M.M.Oberman**

**Macmillan Education**

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MACMILLAN EDUCATION

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*'Counting is  
the oldest art  
of science'*

# *Preface*

During the last decade the number of books on switching has rapidly increased. These books cover for an important portion of their contents the theory and design of logical circuits, thereby using complicated methods which are not required for many applications. In this type of book the design of counter circuits is hardly treated, even though counting is an operation of major importance in nearly any digital automatic circuit and system.

There is hardly any design problem in binary counters running through all possible state combinations of their memory elements. The problem starts with decade counters using a subset of 10-out-of-16 4-bit binary code combinations. A number of these subsets have been used in decade counters. In all of these counters there is the problem of the jump from one code combination to another which is not the following in the binary sequence. Many examples of bad design have been published in literature even after the correct solution of this problem was known.

Some new components are introduced in this text, such as the *A-S* flip-flop and the accumulator. With the *A-S* flip-flop as building stone the construction of some counter circuits is simplified, and with the accumulator nearly all single-chip counter circuits become superfluous because all these different types of counters can be designed using the accumulator as starting point.

The text of this book is limited in that no counter circuits are discussed for very high-speed work as are required in atomic research. The text covers the counters which can be designed with standard digital integrated circuits, and those counters which are commercially available as single-chip integrated circuits. There are already so many different counting ideas in this field that this text has taken on the character of a handbook on electronic counters.

In the design of many digital automatic circuits and systems only those counters are used which are commercially available as single-chip integrated circuits. Without having a book on electronic counters, many circuit designers might think that the switching methods found in these electronic counters are the only practical counting methods. The synchronous binary rate multiplier is a striking example of the contrary.

The text of this book is especially written to give the circuit designer a comprehensive survey of electronic counters. This is the first complete book on this topic.

This book can also be used in the educational field, in switching laboratories of universities and technical institutes for example, to provide them with problems for practical exercises in digital switching. Many of the ideas given in the following chapters have been tested on the switching equipment for the practical exercises in my laboratory, or by means of a program on the PDP 9L computer in my laboratory.

The diagrams in the text have been checked by T. van Onzen and W. G. van den Berg. A. Snijders, F. L. Muller and A. P. Thijssen have contributed to this text with new ideas. I want to express my thanks to them for their assistance and suggestions and I want to extend this to all who have cooperated with me in one or other way in the preparation of this manuscript.

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# 1

## *Drawing symbols, switching functions and diagrams*

### 1.1 Gate symbols

In this text many different ideas on electronic counters will be discussed; they will be expressed in switching functions for the control of flip-flops and expressed in diagrams by means of drawing symbols. Because all the circuits are designed to be implemented by means of integrated circuits or represent a single integrated circuit, a set of drawing symbols has been chosen which are ordinarily used in these circuits. Whether or not this is the most useful set of symbols is not discussed; these symbols are certainly not the easiest to draw.

The first four symbols of figure 1.1 have been indicated as 2-input gates. However, in TTL integrated form some of them are available with a higher number of inputs. These variations have been indicated in the column *package*. AND gates are, for example, commercially available in packages with four 2-input gates, three 3-input gates and two 4-input gates. In the column *function* the switching equations in the 2-input case have been indicated.

The number of inputs of the exclusive-OR gate (and the not commercially available exclusive-NOR gate) is by their nature limited to two inputs. The switching equations of these gates need not be explained here because they belong to the ordinary switching algebra, with which the reader is assumed to be familiar.

Symbols 1.7 and 1.8 represent respectively the more complicated AND-OR gate and the AND-OR-INVERT gate which are not frequently used in this text.

Symbol 1.9 represents a 3-input standard gate, elsewhere proposed by the author,<sup>1</sup> which in fact can be programmed to operate in the NAND, OR, ex.-OR, 0/1, true/complement mode of operation. Moreover, its switching function

$$S = ab' + bc' \quad (1.1)$$

reduces with  $c = 1$  to  $S = ab'$  or with  $a = 0$  to  $S = bc'$ . Because the  $ab'$  function is available, there is little or no need for a standard gate which can also be programmed to perform the AND, NOR, and ex.-NOR functions.

In some of the examples of the following chapters the standard gate will be used notwithstanding the fact that, at the time of writing, it is not commercially available as a (quadruple) single-chip integrated circuit.

The tenth symbol in figure 1.1 is the inverter which needs no further elucidation.

	symbol	name of gate	function	package
1.1		AND	$S = ab$	4x2; 3x3; 2x4
1.2		NAND	$S = (ab)'$	4x2; 3x3; 2x4; 1x8
1.3		OR	$S = a + b$	4x2
1.4		NOR	$S = (a + b)'$	4x2; 3x3; 2x4
1.5		EX.-OR	$S = a \oplus b$	4x2
1.6		EX.-NOR	$S = (a \oplus b)'$	
1.7		AND-OR	$S = ab + cd$	
1.8		AND-OR INVERT	$S = (ab + cd)'$	2x
1.9		STANDARD	$S = ab' + bc'$	4x2
1.10		INVERTER	$S = a'$	6x1

Figure 1.1 Gate symbols

**1.2 Flip-flop symbols**

The second group of symbols (figure 1.2) contains the flip-flops. The symbols of all these flip-flops are based on a rectangle with on one side the information and control inputs and on the other side the information outputs. In flip-flops with more than four terminals, clock pulse

	symbol	name	function
2.1		<i>D</i> flip-flop	$Q_{n+1} = D$
2.2		<i>T</i> flip-flop	$Q_{n+1} = T' Q_n + T Q_n$
2.3		<i>R-S</i> flip-flop	$Q_{n+1} = R' (S + Q_n)$
2.4		<i>J-K</i> flip-flop	$Q_{n+1} = J Q_n' + K' Q_n$
2.5		<i>A-S</i> flip-flop	$Q_{n+1} = A \bullet S Q_n$
2.6		flip-flop with preset and clear	

Figure 1.2 Flip-flop symbols