

DISCRETE MATHEMATICS AND ITS APPLICATIONS

Series Editor KENNETH H. ROSEN

COMBINATORICS OF COMPOSITIONS AND WORDS



SILVIA HEUBACH
TOUFIK MANSOUR

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Für meine Eltern - ihr habt mir die Freiheit gelassen meinen eigenen Weg zu gehen.

Silvia

لَأُمِّي - لِكُونِهَا فِدْوَةٌ لَنَا بِصُمُودِهَا فِي وَجْهِ الصَّعَابِ.

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